

# Capacity Allocation and Pricing Strategies for Wireless Femtocell Services

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Indoor cell phone users often suffer from poor connectivity. One promising solution, femtocell technology, has been rapidly developed and deployed over the past few years. One of the biggest challenges for femtocell deployment is lack of a clear business model. This paper investigates the economic incentive for the cellular operator (also called macrocell operator) to enable femtocell service by leasing spectrum resource to an independent femtocell operator. On the one hand, femtocell services can increase communication service quality and thus increase the efficiency of the spectrum resource. On the other hand, femtocell services may introduce more competition to the market. We model the interactions between a macrocell operator, a femtocell operator, and users as a three-stage dynamic game, and derive the equilibrium pricing and capacity allocation decisions. We show that when spectrum resources are very limited, the macrocell operator has incentive to lease spectrum to femtocell operators, as femtocell service can provide access to more users and efficiently increase the coverage. However, when the total spectrum resource is large, femtocell service offers significant competition to macrocell service. Macrocell operator thus has less incentive to enable femtocell service. We also investigate the issue of additional operational cost and limited coverage of femtocell service on equilibrium decisions, consumer surplus and social welfare.

*Key words:* game theory; simulation: analysis; telecommunications

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## 1. Introduction

Today there are over 5 billion cellphone users in the world (Global mobile statistics 2011). Many users experience poor indoor reception at home or office. This is because in the current cellular network (also called macrocell network), high-frequency and low-power cell signal has

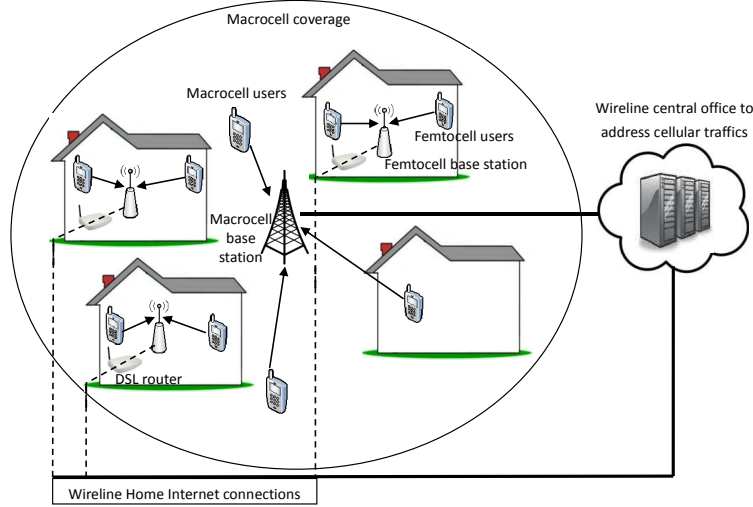


Figure 1: Coexistence of femtocell service and macrocell service, where a macrocell and three femtocells are deployed

to travel between the outside cell site and the indoor cell phones through various obstacles, including brick walls, metal, and even trees, which leads to significant signal attenuations and dropped calls (Sandler 2009).

One promising solution to the indoor reception problem, femtocell technology has been rapidly developed and deployed over the past few years. Femtocells use small base stations of sizes similar to wireless routers. These femtocell base stations are deployed indoors, and can pick up indoor users' mobile signals easily and route calls to the cellular network through home Internet connection. Femtocell technology can significantly increase the quality of voice calls and improve the speed of data communications (Shetty et al. 2009). Figure 1 provides an illustration of four homes with macrocell coverage and three of them have installed femtocell base stations.

Currently in the United States, AT&T, Sprint Nextel and Verizon Wireless (a joint venture of Verizon Communications Inc. and Vodafone Group PLC) are already offering femtocell services to their customers. T-Mobile and Vodafone in Europe, NTT DoCoMo and Softbank in Japan, and Unicom in China have been conducting tests of the technology and planning to roll out nationwide femtocell services. In June 2010, UK research firm Informa Telecoms & Media reported that femtocell deployments had more than doubled in the past 12 months, with more and more tier one operators jumping on the bandwagon (Informa Telecoms & Media 2011). Shipments are estimated to grow from 0.2 million units in 2009 to 12 million units worldwide in 2014 (Berg Insight 2009).

However, one of the biggest challenges to companies' wide femtocell deployment is the lack of a clear business model. As Emin Gurdenli, chief technology officer of Deutsche Telekom AG's T-Mobile U.K., put it (The Wall Street Journal, Feb. 2009):

*“The rationale for femtocells is well-established, but a quantitative business case with a clear business model in terms of how we go to market is not there yet.”*

The purpose of this paper is to develop such a quantitative model to examine the trade-off regarding femtocell deployment. In particular, we look at the following research questions:

- *Should current macrocell operators deploy femtocell services? How would operators allocate bandwidth (capacity) resources and make pricing decisions?* There are two common approaches to the deployment of femtocell service. In an *integrated* system, a macrocell operator directly provides femtocell service to users and fully controls bandwidth resource allocation and femtocell service price. We also submitted a paper on the economic operation of integrated system (Duan et al. 2011). In a *distributed* system, a macrocell operator leases its spectrum resources to a femtocell operator. The femtocell operator determines the service provision and pricing independently. We can find many such examples in industry: Sprint leases licensed spectrum to Virgin Mobile USA to provide femtocell service (Fitchard 2009), and BT Mobile is using Vodafone's resource to provide femtocell service (Atkinson 2011). Recently there are more research on the distributed system (e.g., Hong and Tsai 2010 and Chen et al. 2011), and this paper focuses on the distributed system. The key tradeoff for the macrocell operator is obtaining more revenue by leasing resources to the femtocell operator as against having fewer resources for its own services and facing increased market competition.
- *How would users choose between femtocell and macrocell services?* With the deployment of indoor femtocell stations, femtocell users no longer experience signal attenuation and poor reception problem, and can achieve the maximum quality of service. In contrast, when users are connected to the outdoor macrocell base stations, the quality of service highly depends on the user locations and the communication environments. When different qualities of services are coupled with different pricing schemes, different users have different preferences between macrocell and femtocell services.

Our main results are summarized as follows:

- *Characterization of equilibrium decisions:* We derive the threshold of spectrum efficiency level which segments users who prefer femtocell to macrocell services. Furthermore, we characterize the femtocell operator's equilibrium femtocell price and the macrocell operator's capacity allocation and pricing decisions.
- *Analysis of impact of macrocell's total limited capacity:* Wireless spectrum is a very scarce resource so macrocell operators often face capacity constraints. In the U.S. 700MHZ spectrum auction in March 2008, the total bid price is nearly \$20b (WNN Wi-Fi Net News 2008). We show that macrocell operator has more incentive to lease spectrum to the femtocell operator when its capacity is *small*, but chooses to offer only macrocell service when its capacity is *large*.
- *Calculation of consumer surplus and social welfare:* With no additional operational cost and full coverage, femtocell service can increase both the *total* consumer surplus and social welfare. However, we show that some users might experience a smaller payoff from the adoption of the femtocell service if, for example, they do not experience much service quality improvement with the femtocell service but need to pay a higher price.

In addition, we have examined two extensions of the basic model. The first is with additional femtocell operational cost. Although femtocells are low in *deployment* costs, the femtocell service may incur additional *operational* cost compared to macrocell service. For instance, femtocell operators may be charged by internet service providers for routing traffic through wireline broadband internet to reach the cellular network (McKnight et al. 1997). The impact of the additional operator cost on the femtocell operator is obviously negative; its impact on the macrocell operators, however, is unclear and deserves detailed exploration. The second is the impact of limited femtocell coverage. A femtocell base station typically has a smaller spatial coverage. For instance, a femtocell device may only cover a region with a diameter of 50-100 meters, whereas a macrocell covers a larger range with a diameter of more than 10 kilometers. The femtocell service may have limited coverage when it does not have enough femtocell base stations. We examine the impact of such limited coverage on macrocell and femtocell operators' profits.

The rest of the paper is organized as follows. We introduce the network model of macrocell service in Section 3, which serves as a benchmark for later analysis. In Section 4, we introduce the network model of femtocell service and analyze how the macrocell operator

and femtocell operator make capacity and pricing decisions to maximize their own profits. Then, in Sections 5 and 6, we extend the results in Section 4 by examining the various effects of femtocell operational cost and limited femtocell coverage. In Section 7 we present the conclusion to our study and discuss future work.

## 2. Literature Review

Our work is closely related to two main streams of literature: i) studies of femtocell deployment in the telecommunication literature, and ii) studies of dual channel competition in the management science and operations research literature.

Most existing work on femtocell deployment in the telecommunication literature (*e.g.*, Chandrasekhar and Andrews 2009) focus on various technical issues in service provision such as access control, resource management, and interference management. Only a few papers discuss the economic issues of femtocells (*e.g.*, Claussen et al. 2007, Yun et al. 2011, Shetty et al. 2009, Chen et al. 2011), examining the impact of network deployment costs and femtocells' openness to macrocell users. The key difference between our paper and such existing literature is that we study the provision of dual services in terms of both spectrum allocations and pricing decisions. We also characterize the impact of the femtocell operational cost and limited femtocell coverage on the service provision.

Our work is also closely related to the literature on *dual channel competition* in the area of management science and operations research. In this body of literature, there are usually two types of decision makers: a manufacturer and a retailer. The manufacturer can sell the products through a direct channel, a retailer channel, or both. Chiang et al. study whether and how a manufacturer should operate a new direct channel when it already has a retailer partner. They show that direct marketing can indirectly increase the flow of profits through a retail channel by reducing the degree of double marginalization. Also, the direct channel may not be a threat to the retailer since the wholesale price is driven down. Tsay and Agrawal 2004 further exploit several means whereby the manufacturer can mitigate channel conflict between the direct channel and the retailer channel, including adjustments of wholesale price, paying a commission to a retailer, and entirely conceding demand fulfillment on the part of the retailer. More general results are obtained motivated by the models in Chiang et al. 2003 and Tsay and Agrawal 2004. For example, Huang and Swaminathan 2009 posit a stylized deterministic demand model where each channel relies on prices, degree of substitution across

channels, and the overall market potential. Dumrongsiri et al. 2008 investigate the influence of demand variability on prices and manufacturer's incentive to open direct channel.

In the context with which we are concerned, that of femtocell deployment, we can view the *macrocell operator* as the manufacturer, the *femtocell operator* as the retailer, and the *macrocell service* as the direct channel. Our paper has four key differences from prior literature.

First, we consider a different order of introducing the new channel. Instead of introducing the direct channel after the retailer channel, as is the case in Chiang et al. 2003, Tsay and Agrawal 2004a, Huang and Swaminathan 2009, Dumrongsiri et al. 2008, we consider the case in which the manufacturer owns the direct channel first and decides on the best way to open the retailer channel.

Second, the limited capacity model considerably complicates the analysis of our model. The dual channel literature generally assumes unlimited potential supply, *i.e.*, that the manufacturer can produce as many products as possible (with a production cost) to maximize its profit. However, a macrocell operator often has only a limited total capacity in the decision time scale considered here. This is because the spectrum allocation to cellular service providers are often regulated by government authorities (*e.g.*, FCC in USA and Ofcom in UK). The macrocell operator often obtains spectrum licenses that last for years or decades. The long license period ensures enough motivation for the macrocell providers to invest in the necessary network infrastructure, which is often very expensive.

Third, the heterogeneity of users in our model is motivated by the unique characteristics of wireless communications, and is thus different from that considered by prior literature. In particular, users have different channel conditions (and thus different evaluations of the same resource allocation) under the macrocell service (direct channel), but have the same maximum channel condition under the femtocell service (retailer channel). In contrast, prior literature either assumes that users are homogenous or are different in willingness to pay. Moreover, the users' utility functions here are also motivated by today's wireless communication technologies, which renders some of the prior generic analysis inapplicable.

Finally, we characterize the impact of limited femtocell coverage on the new service provision. Very few prior studies have considered a similar constraint. Rubin 1978 considers a related constraint where the monitoring costs of company-owned outlets rise with physical distance from headquarters, and thus direct channel becomes non-profitable in suburban areas. What we considered is the limitation of coverage of the retailer channel, and the

model thus is different.

### 3. Benchmark Scenario: Macrocell Service Only

Throughout this paper, we focus on the monopoly case in the two-tier market with a single macrocell operator and a single femtocell operator. The reason is that we can observe many monopoly examples in macrocell service worldwide (e.g., America Movil (the world's fourth largest mobile network operator in Mexico and many places in Latin America), and MTS in some central Asian countries). Also, since femtocell service just emerged from last decade, many femtocell operators are still local monopolists (e.g., Virgin Mobile USA in US and BT Mobile in UK). Moreover, monopoly is a first step leading to a more general oligopoly market and we plan to study oligopoly case as a future direction as in Section 7.

As a benchmark case, we first look at how the macrocell operator prices the macrocell service to maximize its profit without introducing the femtocell service. When we consider the introduction of femtocell service in Sections 4 and 5, the macrocell operator needs to achieve a profit no worse than this benchmark case.

For the sake of discussion, we will focus on the operation of a single macrocell. In general, a macrocell operator owns multiple macrocells. Non-adjacent macrocells can share the same frequency (called frequency reuse). The analysis of this paper can be extended to the more general case without changing the main managerial insights.

The macrocell operator owns wireless spectrum (also called bandwidth) with a limited capacity; a user needs to access the bandwidth in order to complete its wireless communications (e.g., voice calls, video streaming, data transfer). A larger bandwidth means more resources to the user and thus better communication quality of service (QoS), but also leads to a greater expense.

As shown in Fig. 2, we model the interactions between the macrocell operator and end users as a two-stage Stackelberg game. In Stage I, the macrocell operator determines the macrocell price  $p_M$  per unit bandwidth. In Stage II, each user decides how much bandwidth to purchase. The operator wants to maximize its profit, while the users want to maximize their payoff. Such usage-based pricing scheme is widely used in today's cellular macrocell networks, especially in Europe and Asia (Courcoubetis and Weber 2003, Altmann and Chu 2001). In US, AT&T (since a year ago) and Verizon (since July 2011) have adopted the usage-based pricing for wireless data services. Usage-based pricing for femtocells has just

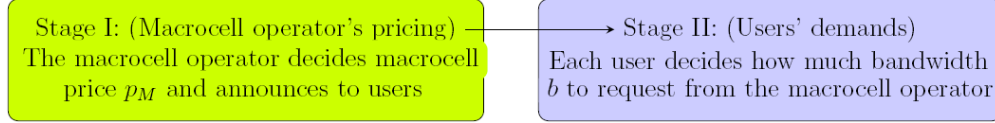


Figure 2: Two-stage Stackelberg game between the macrocell operator and users.

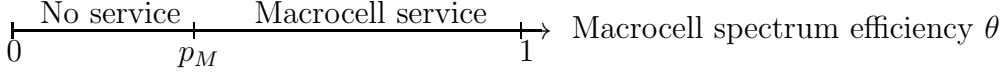


Figure 3: Distribution of users' macrocell spectrum efficiency  $\theta$

started. For example, AT&T's femtocell service counts the femtocell data usage as part of the regular cellular usage (together with the macrocell data usage), which is subject to usage-based pricing (AT&T 2011). Due to the exponential growth of wireless data traffic and the scarce spectrum resource, we envision that usage-based pricing for both macrocell and femtocell services will become more and more common in the near future.

Next, we solve this two-stage Stackelberg game by backward induction (Myerson 1997).

### 3.1 Stage II: Users' Bandwidth Demand Given A Fixed Price $p_M$

The QoS of a wireless communication session depends not only on the resource allocation but also on the condition of the wireless channel between the transmitter and receiver. The channel condition is determined by both the locations of transmitter and receiver and the surrounding environment. As an example, let us consider uplink transmissions from the users mobile phones to the common single macrocell base station (as in Fig. 1). The channel condition in general decreases with the distance between the user and the base station, and can become very weak if the user is inside a house with thick walls. A user with a bad channel condition will not be able to achieve a high data rate even with a large bandwidth allocation.

Here we model the users' channel heterogeneity by a *macrocell spectrum efficiency*  $\theta$ , which is assumed to be uniformly distributed in  $[0, 1]$  (see Fig. 3). The uniform distribution is assumed for analytical tractability. A more complicated distribution based on field measurements will not change the main managerial insights obtained in this paper. A larger  $\theta$  means a better channel condition and a higher spectrum efficiency *when using the macrocell service*.

For a user with a macrocell spectrum efficiency  $\theta < 1$ , when allocated macrocell band-



width  $b$ , its effective resource allocation is  $\theta b$ . Its *utility*  $u(\theta, b)$  (e.g., data rate) can be modeled as (similar to Sengupta and Chatterjee 2009, Wang and Li 2005)

$$u(\theta, b) = \ln(1 + \theta b),$$

which is concave in  $b$  representing the diminishing return in bandwidth consumption. The more bandwidth a user obtains, he can experience a higher data rate and a better QoS when communicating with others. The user needs to pay a linear payment  $p_M b$  to the macrocell operator, where the price  $p_M$  is announced by the macrocell operator in Stage I. Note that the usage-based pricing is becoming a main trend in macrocell service market (and replacing flat-fee pricing for data traffics) (Goldstein 2011). The user's *payoff* is the difference between the utility and payment, *i.e.*,

$$r_M(\theta, b, p_M) = \ln(1 + \theta b) - p_M b. \quad (1)$$

The optimal value of bandwidth (demand) that maximizes the user's payoff with the macrocell service is

$$b^*(\theta, p_M) = \frac{1}{p_M} - \frac{1}{\theta} \quad (2)$$

if  $p_M \leq \theta$  and 0 otherwise. Notice that  $b^*(\theta, p_M)$  is decreasing in  $p_M$  and increasing in  $\theta$  (if  $p_M \leq \theta$ ). When  $p_M > \theta$ , the user chooses not to start the wireless communication as it is too expensive (by taking its macrocell spectrum efficiency  $\theta$  into consideration). The user's maximum payoff with macrocell service is

$$r_M(\theta, b^*(\theta, p_M), p_M) = \ln\left(\frac{\theta}{p_M}\right) - 1 + \frac{p_M}{\theta} \quad (3)$$

if  $p_M \leq \theta$  and 0 otherwise. Notice that the payoff is always nonnegative.

### 3.2 Stage I: Macrocell Operator's Pricing $p_M$

Next we consider the macrocell operator's optimal choice of price  $p_M$  in Stage I. To achieve a positive profit, the macrocell operator needs to set  $p_M \leq \max_{\theta \in [0,1]} \theta = 1$ , otherwise no user will request any bandwidth in Stage II.

Without loss of generality, we normalize the total user population to 1. The fraction of users choosing macrocell service is  $1 - p_M$  as shown in Fig. 3. The total user demand is

$$Q_M(p_M) = \int_{p_M}^1 \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta = \frac{1}{p_M} - 1 + \ln p_M, \quad (4)$$

which is a decreasing function of  $p_M$ .

Recall that the macrocell operator has a limited bandwidth capacity  $B$ , and thus can only satisfy a total demand no larger than  $B$ . The macrocell operator's profit is

$$\pi^{macro}(p_M) = p_M \min \left( B, \frac{1}{p_M} - 1 + \ln p_M \right).$$

The operator will choose price  $p_M$  to maximize profit, *i.e.*,

$$\max_{0 < p_M \leq 1} \pi^{macro}(p_M). \quad (5)$$

Theorem 1 characterizes the unique optimal solution to Problem (5).

**Theorem 1.** *The equilibrium macrocell price  $p_M^{bench}$  that maximizes the macrocell operator's profit in the two-stage Stackelberg game in Fig. 2 is the unique solution to the following equation:*

$$B = \frac{1}{p_M} - 1 + \ln p_M. \quad (6)$$

*The total user demand equals the maximum capacity at the equilibrium, i.e.,  $Q_M(p_M^{bench}) = B$ . The equilibrium price  $p_M^{bench}$  decreases with  $B$ , and the macrocell operator's equilibrium profit  $\pi^{macro}(p_M^{bench})$  increases with  $B$ .*

Notice that no users with a macrocell spectrum efficiency  $\theta$  less than  $p_M^{bench}$  will receive macrocell service. When the total bandwidth  $B$  is small, the equilibrium macrocell price  $p_M^{bench}$  is close to 1 and thus most users will not get service. This motivates the macrocell operator to adopt the femtocell service, which is able to serve these users and leads to additional profit, which is shared by the macrocell and femtocell operators.

## 4. Femtocell Deployment

We now turn to the question of how femtocell service may improve the macrocell operator's profit. We are interested in understanding the following issues:

- *Strategic decision:* Is it economically beneficial for the macrocell operator to lease spectrum to a femtocell operator, who will compete with the macrocell operator in serving the same group of end users?
- *Operational decisions:* If the answer to the previous question is yes, how should the macrocell operator allocate and price the spectrum resources?

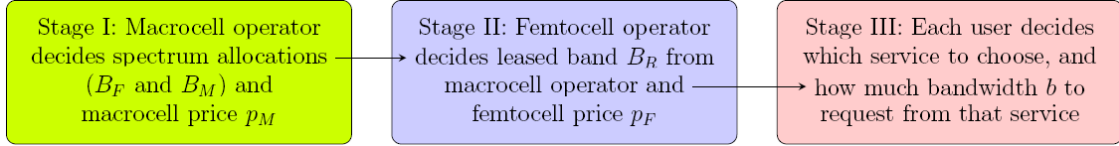


Figure 4: Three-stage dynamic game between the macrocell operator, femtocell operator, and users.

The analysis in this section is based on several simplified assumptions:

- The femtocell service does not incur any additional operational cost compared to the macrocell service. This assumption will be relaxed in Section 5.
- The femtocell service has the same coverage as the macrocell service, such that each user has the choice between two services. This assumption will be relaxed in Section 6.

It should be noted that if we simultaneously relax both the two assumptions later, it is difficult to see the impact of each explicitly. Thus we relax them in separate sections.

More specifically, we will look at a three-stage dynamic game as in Fig. 4. The macrocell operator has market power and is the leader in the cellular market, while the emerging femtocell operator is the follower. In Stage I, the macrocell operator decides bandwidth allocations to femtocell service  $B_F$  and macrocell service  $B_M$  such that  $B_F + B_M = B$ . Here we focus on “separate carriers” scheme where dual services operate on independent spectrum bands. “Separate carriers” is easy to manage and can avoid interferences between macrocells and femtocells. For example, China Unicom (one of the top 3 wireless service providers in China and the first one deploying femtocell since 2009) is in strong favor of this scheme (China Femtocell Symposium 2011). There exists another scheme called “shared carriers” where dual services operate on the same spectrum bands, which is discussed in Section 7. The macrocell operator also decides the macrocell price  $p_M$ , which is charged to both the femtocell operator and end users who choose macrocell services. Note that we assume the same price is charged to both femtocell operator and users of macrocell services, to avoid arbitrage opportunity. For example, if the macrocell operator charges the femtocell operator more than macrocell users, the femtocell operator can pretend to be macrocell users, request spectrum at the macrocell price, and serve its femtocell users. Such an approach has been studied in Tanneur 2003. If the macrocell operator charges the femtocell operator less than macrocell users, then some other intermediate operator, e.g., mobile virtual network operators, can disguise themselves as femtocell operators and obtain spectrum resource at

a lower price than macrocell price, and then provide macrocell service to users to make a profit. This eventually decreases the macrocell operator's market share and profit. Such scenarios have been considered in Dewenter and Haucap 2006.

In Stage II, femtocell operator decides how much bandwidth  $B_R$  to lease from the macrocell operator such that  $B_R \leq B_F$ , and pay  $B_R p_M$  to the macrocell operator. It also determines femtocell price  $p_F$  to end users choosing the femtocell service. In Stage III, each user decides which service to choose and how much bandwidth to purchase for the service. If a user's preferred service is not available (when demand is larger than capacity for that service), the user will seek the other service. Here we focus on a large group of users, where a single user's demand is infinitesimal to the total demand. Thus we can ignore cases in which a user purchases bandwidth from both services.

Our considered decision process in Stages I and II is like the existing spectrum auction, where the spectrum holder announces total bandwidth amount and price first, then bidders request and give out payment. There may possibly exist some other decision process, e.g., femtocell operator requests some bandwidth from macrocell operator first and then macrocell operator decides how much to satisfy at some price. Intuitively, these two processes should lead to the same equilibrium outcome. This is because the macrocell operator can adjust macrocell price to make its decided femtocell band matches femtocell operator's demand. It should also be noted that it is optimal for the femtocell operator to announce femtocell price after purchasing bandwidth from the macrocell operator. Otherwise, it may run out of bandwidth at a low price or waste some bandwidth.

We will again analyze this three-stage dynamic game using backward induction.

To differentiate from the macrocell service only benchmark in Section 3, we refer to the setup in this section as *dual services*. Notice that dual services may degenerate to the case of benchmark when the macrocell operator decides not to lease spectrum to the femtocell operator, *i.e.*, when the total capacity is large as shown later in this section.

#### 4.1 Stage III: Users' Service Choice and Bandwidth Demand

If a user has a macrocell spectrum efficiency  $\theta$ , his optimal payoff by using the macrocell service is given in (3). Next, we consider a user's payoff by using the femtocell service.

Since femtocell base stations are deployed indoors and are very close to users' cell phones, it is reasonable to assume that all users using the femtocell service have equally good channel

conditions and achieve the same *maximum* spectrum efficiency. This means that, independent of the macrocell spectrum efficiency  $\theta$ , each user achieves the same *payoff*  $r_F(b, p_F)$  when using a bandwidth of  $b$  under femtocell service,

$$r_F(b, p_F) = \ln(1 + b) - p_F b. \quad (7)$$

The user's optimal demand in femtocells is

$$b^*(p_F) = \frac{1}{p_F} - 1 \quad (8)$$

if  $p_F \leq 1$  and 0 otherwise. A user's maximum payoff under the femtocell service is

$$r_F(b^*(p_F), p_F) = \ln\left(\frac{1}{p_F}\right) - 1 + p_F \quad (9)$$

if  $p_F \leq 1$ , and 0 otherwise. which is always nonnegative. Note that some operators have adopted a flat-fee charging scheme for femtocell services to encourage early user adoptions. In this paper, we focus on analyzing the usage-based pricing for femtocell services in a mature market. It should also be noted that usage-based pricing often leads to a higher profit than the flat-fee charging (Courcoubetis and Weber 2003).

We will show that  $p_F > p_M$  at the equilibrium, *i.e.*, the femtocell price  $p_F$  in Stage II, is always larger than the macrocell price  $p_M$  in Stage I. By comparing the user's payoffs in (3) and (9), it is clear that a user with  $\theta = 1$  will always choose macrocell service to maximize his payoff. On the other hand, a user with a small  $\theta$  would choose femtocell service to improve his payoff. As a result, we define the following thresholds of  $\theta$  that separate the user population into two service groups.

**Definition 1** (Users' preferred partition threshold  $\theta_{th}^{pr}$ ). *Users with  $\theta \in [0, \theta_{th}^{pr})$  prefer to use the femtocell service, and users with  $\theta \in [\theta_{th}^{pr}, 1]$  prefer to use the macrocell service.*

**Definition 2** (Users' finalized partition threshold  $\theta_{th}$ ). *The finalized partition threshold  $\theta_{th}$  is the minimum macrocell spectrum efficiency among all the users actually served by the macrocell service. Users with  $\theta \in [\theta_{th}, 1]$  receive the macrocell service, while users with  $\theta \in [0, \theta_{th})$  receive either the femtocell service or no service.*

The preferred partition threshold  $\theta_{th}^{pr}$  only depends on prices  $p_M$  and  $p_F$ . If all users' demands from their preferred services are satisfied, then users' preferred partition threshold equals users' partition threshold (*i.e.*,  $\theta_{th}^{pr} = \theta_{th}$ ). However, in general  $\theta_{th}$  may be different from  $\theta_{th}^{pr}$ , depending on the values of  $B_F$ ,  $B_M$ , and  $B_R$  in Stages I and II.

By comparing a user's optimal payoff with macrocell and femtocell services in (3) and (9), we obtain the following result.

**Lemma 1.** *Users' preferred partition threshold  $\theta_{th}^{pr} = p_M/p_F$ .*

Now we introduce the concept of *finalized demand*.

**Definition 3** (User's Finalized Demand). *If a user's demand from his preferred service is satisfied, then his finalized demand equals his preferred demand. If not, the user switches to the alternative service and the new demand becomes the finalized demand.*

Note that a user's finalized demand may not be realized, *e.g.*, when the price is set too low and the total finalized demand is larger than the supply.

## 4.2 Stage II: Femtocell Operator's Spectrum Purchase and Pricing

Now we analyze Stage II, where the femtocell operator determines  $B_R$  and  $p_F$  to maximize its profit. In this stage, the macrocell operator's decisions on  $p_M$  and  $B_F$  (and  $B_M = B - B_F$ ) are determined and known to the femtocell operator. Let us denote the femtocell operator's equilibrium decisions as  $B_R^*(p_M, B_F)$  and  $p_F^*(p_M, B_F)$ , both of which are functions of  $p_M$  and  $B_F$ .

To maximize profit, the femtocell operator needs to know which users will choose femtocell service and their characteristics. Users with macrocell spectrum efficiency  $\theta \in [0, \theta_{th}^{pr}] = [0, p_M/p_F)$  will choose femtocell service first. Some other users may also choose femtocell service if their demands cannot be satisfied by the macrocell services. The following lemma, however, shows that the macrocell operator will reserve enough bandwidth  $B_M$  during Stage I, such that all users who prefer to use macrocell service will be able to do so.

**Lemma 2.** *At the equilibrium of the three-stage dynamic game as in Fig. 4, the macrocell operator satisfies all preferred demands from users with  $\theta \in [\theta_{th}^{pr}, 1] = [p_M/p_F, 1]$  in macrocell service.*

Lemma 2 is derived regardless of the decisions of femtocell operator. Thus at the equilibrium of the whole three-stage game, Lemma 2 holds and we can use it to derive femtocell operator's equilibrium decisions in Stage II.

We will further discuss the intuitions of Lemma 2 in the next subsection. Since femtocell operator only serves users with  $\theta \in [0, p_M/p_F)$ , its profit is

$$\begin{aligned}\pi^{Femto}(p_F, B_R) &= p_F \min \left( B_R, \int_0^{\frac{p_M}{p_F}} \left( \frac{1}{p_F} - 1 \right) d\theta \right) - p_M B_R \\ &= \min \left( (p_F - p_M) B_R, (1 - p_F) \frac{p_M}{p_F} - p_M B_R \right).\end{aligned}\quad (10)$$

The femtocell operator's profit-maximization problem is:

$$\begin{aligned}\max_{p_F \geq 0, B_R \geq 0} \quad & \pi^{Femto}(p_F, B_R) \\ \text{subject to} \quad & B_R \leq B_F.\end{aligned}\quad (11)$$

By solving Problem (11), we have the following result.

**Lemma 3.** *In Stage II, the femtocell operator's equilibrium femtocell price is*

$$p_F^*(p_M, B_F) = \max \left( \frac{2p_M}{1 + p_M}, \frac{-p_M + \sqrt{(p_M)^2 + 4B_F p_M}}{2B_F} \right), \quad (12)$$

*and its equilibrium femtocell bandwidth purchase is*

$$B_R^*(p_M, B_F) = \min \left( \frac{1 - (p_M)^2}{4p_M}, B_F \right), \quad (13)$$

*which equals users' total preferred demand in femtocell service. Then users' preferred partition threshold equals equilibrium partition threshold (i.e.,  $\theta_{th}^{pr} = \theta_{th}$ ).*

The proof of Lemma 3 is given in Appendix 1. Lemma 3 shows that the femtocell operator will also satisfy the users' preferred demands, and it does not want the users to switch to its competitor (i.e., the macrocell operator).

### 4.3 Macrocell Operator's Spectrum Allocations and Pricing in Stage I

Now we come back to Stage I, where the macrocell operator determines  $p_M$ ,  $B_F$ , and  $B_M$  to maximize its profit. Let us denote the macrocell operator's equilibrium decisions as  $p_M^*$ ,  $B_F^*$ , and  $B_M^*$ .

Notice that Lemma 2 shows that it is optimal for the macrocell operator to serve all users with  $\theta \in [p_M/p_F^*(p_M, B_F), 1]$  by macrocell service, where  $p_F^*(p_M, B_F)$  is the equilibrium femtocell price given in Lemma 3. This means that the macrocell operator does not want

users with large macrocell spectrum efficiency  $\theta$  to choose its competitor (*i.e.*, the femtocell operator). Intuitively, users with a large  $\theta$  demand more bandwidth in macrocell service than in femtocell service, and thus lead to a larger profit to the macrocell operator if they choose macrocell service.

Since  $B_M = B - B_F$ , we can write the macrocell operator's profit as a function of  $p_M$  and  $B_F$ , *i.e.*,

$$\pi^{Macro}(p_M, B_F) = p_M B_R^*(p_M, B_F) + p_M \int_{\frac{p_M}{p_F^*(p_M, B_F)}}^1 \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta. \quad (14)$$

The macrocell operator's profit-maximization problem is

$$\begin{aligned} & \max_{p_M \geq 0, B_F \geq 0} \quad \pi^{Macro}(p_M, B_F) \\ & \text{subject to} \quad \int_{\frac{p_M}{p_F^*(p_M, B_F)}}^1 \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta \leq B - B_F, \end{aligned} \quad (15)$$

where  $p_F^*(p_M, B_F)$  and  $B_R^*(p_M, B_F)$  are given in (12) and (13), respectively. The constraint shows that macrocell band  $B_M = B - B_F$  can satisfy users' total preferred macrocell demand.

Problem (15) is not convex and is difficult to solve in closed-form, but can be solved easily using numerical methods. Next we introduce a useful lemma that facilitates our later discussions on numerical results.

The macrocell operator wants to sell its total capacity  $B$  at the highest macrocell price  $p_M$ . Under a fixed price, the total demand from the users depends on which services they subscribe to. If we can maximize the user demand under any fixed price, then we can achieve the maximum revenue by optimizing the choice of price accordingly.

Recall that a user's demand is  $\frac{1}{p_F^*(p_M, B_F)} - 1$  in femtocell service and  $\frac{1}{p_M} - \frac{1}{\theta}$  in macrocell service. We have the following lemma.

**Lemma 4.** *The macrocell operator wants users with  $\theta \in \left[ 0, \frac{1}{\frac{1}{p_M} - \frac{1}{p_F^*(p_M, B_F)} + 1} \right)$  to choose femtocell service, and rest of the users with  $\theta \in \left[ \frac{1}{\frac{1}{p_M} - \frac{1}{p_F^*(p_M, B_F)} + 1}, 1 \right]$  to choose macrocell service. That is, it prefers users' partition threshold to be  $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M} - \frac{1}{p_F^*(p_M, B_F)} + 1}$ .*

Note that the threshold in Lemma 4 is what the macrocell operator wants to see; however, it may not equal the users' finalized partition threshold  $\theta_{th} = \frac{p_M}{p_F^*(p_M, B_F)}$ . This is because that the macrocell operator cannot fully control the femtocell operator's decisions. The difference between these two thresholds are due to the market competition between macrocell and femtocell operators.



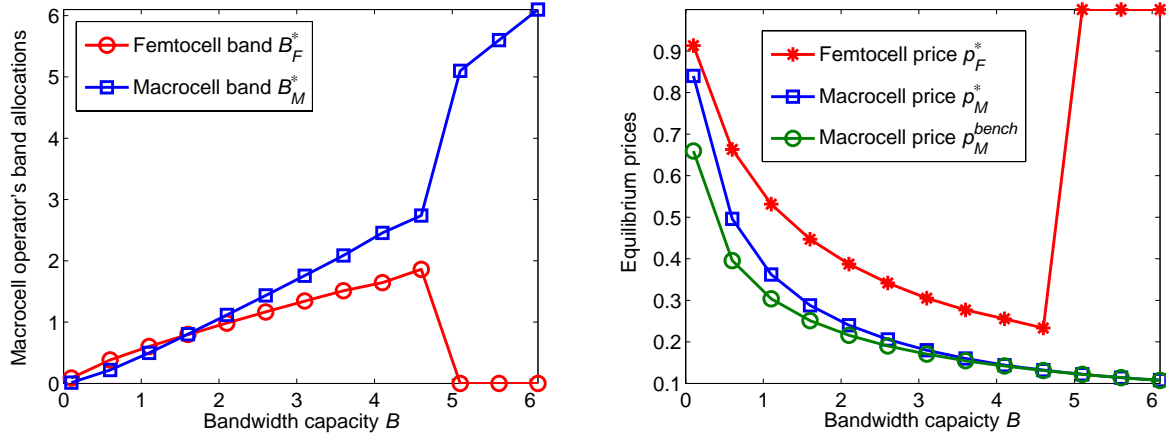


Figure 5: Equilibrium bands  $B_F^*$  and  $B_M^*$  as functions of capacity  $B$  Figure 6:  $p_F^*$  and  $p_M^*$  in dual services, and benchmark price  $p_M^{bench}$  as functions of  $B$

#### 4.4 Numerical Results

Solving Problem (15) numerically, we obtain the macrocell operator's equilibrium femtocell band  $B_F^*$ , and macrocell band  $B_M^* = B - B_F^*$ , and the macrocell price  $p_M^*$ . Plugging into (12), we obtain the equilibrium femtocell price  $p_F^*$ .

Figure 5 shows the macrocell operator's equilibrium bandwidth allocation. The X axis is total bandwidth capacity  $B$  and the Y axis is the macrocell and femtocell bandwidth  $B_F^*$  and  $B_M^*$ , respectively. It shows that when the total bandwidth capacity  $B$  is small, the macrocell operator would lease spectrum to the femtocell operator, so both macrocell and femtocell services are provided to end users; however, when the total bandwidth capacity  $B$  becomes large, only macrocell service is provided (i.e.,  $B_F^* = 0$  and  $B_M^* = B$ ). The intuition behind this is as follows: with large bandwidth capacity, the macrocell operator can already serve most users by macrocell service. The potential benefit of reaching the remaining small portion of customers through facilitating femtocell service can not compensate the potential loss due to new market competition. Hence the macrocell operator has no motive to lease spectrum to femtocell provider. However, with small capacity  $B$ , the macrocell service price  $p_M^*$  is high, and thus most users with  $\theta \in [0, p_M^*)$  would not request macrocell service. By leasing bandwidth to femtocell operator, the macrocell operator can obtain a larger profit from serving more users (indirectly through femtocell operator). It should be noted that  $B = 4.77$  (that distinguishes total capacity to be small or large) is a normalized value compared to users' population, as we have normalized users' population to be 1.

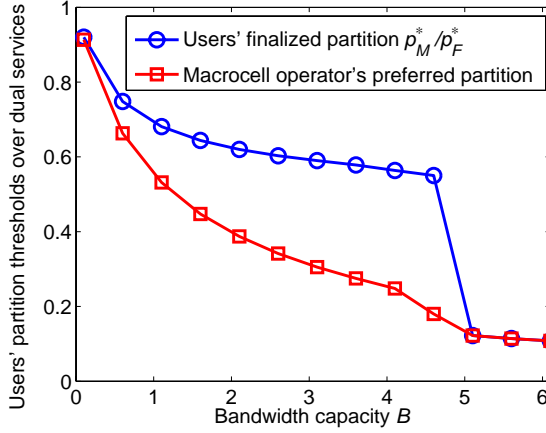


Figure 7: Users' equilibrium partition thresh-  
old  $\theta_{th} = \frac{p_M^*}{p_F^*}$  and macrocell operator's pre-  
ferred  $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*} - \frac{1}{p_F^*} + 1}$  as functions of  $B$

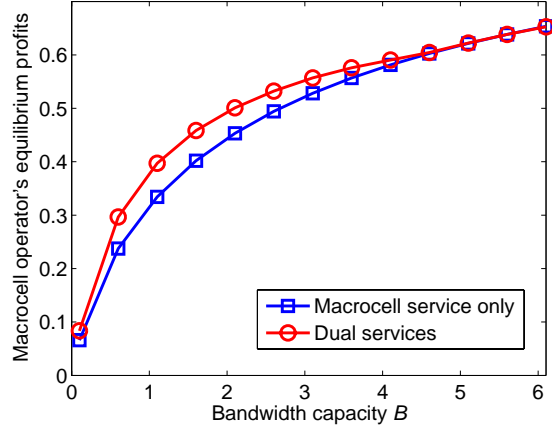


Figure 8: Macrocell operator's profits in  
macrocell service only case and current dual  
service case as functions of capacity  $B$

Figure 6 shows how the femtocell and macrocell prices  $p_F^*$  and  $p_M^*$  change in the total bandwidth capacity. It also shows the macrocell price of the benchmark case  $p_M^{bench}$  where there is macrocell service only. First, we observe that when the total bandwidth capacity  $B$  becomes large, the femtocell price  $p_F^*$  becomes 1 and  $p_M^* = p_M^{bench}$ . This essentially means that only macrocell service is provided, which is consistent with the observation from Fig. 5. Second, the equilibrium macrocell price  $p_M^*$  is always no less than the benchmark price  $p_M^{bench}$ . This means that the macrocell operator can obtain a larger profit with femtocell deployment by reaching more users.

Figure 7 shows users' finalized partition threshold  $\theta_{th} = \frac{p_M^*}{p_F^*}$ , and compares with the threshold  $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*} - \frac{1}{p_F^*} + 1}$  that macrocell operator prefers. It shows that the gap increases in the total capacity  $B$ . This means that femtocell operator attracts more original macrocell users to femtocell service, and competition between two operators becomes more intense as  $B$  increases.

By summarizing the results in Figs. 5 to 7, we have the following result.

**Observation 1.** *Only when its total bandwidth capacity  $B$  is small, the macrocell operator will lease spectrum to the femtocell operator to provide femtocell service and thus serve more users. When  $B$  is large, the macrocell operator will not lease any spectrum to eliminate significant competition from femtocell operator.*

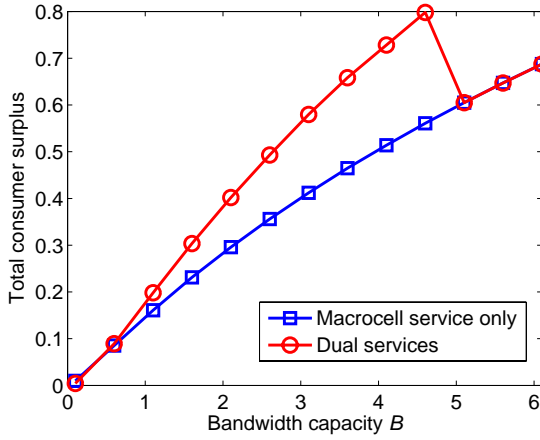


Figure 9: Comparison of total consumer surplus plus between dual services and macrocell service only benchmark as functions of  $B$ .

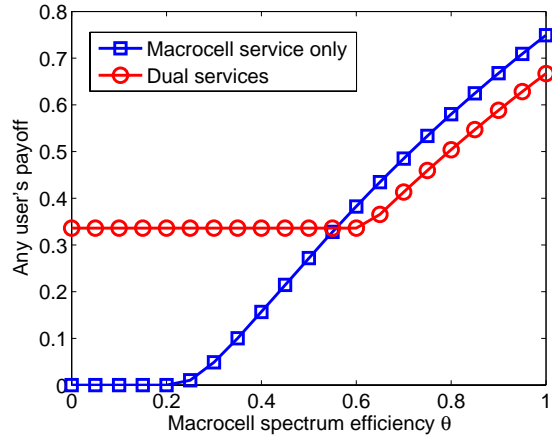


Figure 10: Comparison of a consumer's payoff between dual services and macrocell service only benchmark as functions of  $\theta$ . Here we fix  $B = 2.1$ .

Today's macrocell operators have “small” capacities in big cities, in the sense that their capacities are not enough to match the too many people's sharp growth of wireless data demand. For example, we can witness poor service coverage of AT&T's macrocell service in New York City and San Francisco due to lack of spectrum resource (LaVallee 2009, WNN Wi-Fi Net News 2008). The absolute value of total capacity is small and we have small  $B \leq 4.77$  in these cities (WNN Wi-Fi Net News 2008). Thus we can observe macrocell operators' strong incentives to lease spectrum to femtocell operators in big cities (e.g., Sprint to Virgin Mobile USA and Vodafone to BT Mobile) to serve more users. However, in many other places with fewer user density, femtocell services haven't been deployed yet.

Next we investigate how the introduction of femtocell service affects the macrocell operator's profit, consumer surplus (*i.e.*, users' aggregate payoff), and the social welfare (*i.e.*, summation of the profits of both operators and the payoffs of all users). In each figure, we compare the dual services with the macrocell service only benchmark. Our discussions focus on the dual-service region.

Figure 8 shows the profits of the macrocell operator when both services are provided vs. when only macrocell service is provided, both of which are increasing in capacity  $B$ . The provision of femtocell service improves the macrocell operator's profit significantly, especially when  $B$  is small (e.g., 27% when  $B = 0.1$ ).

Figure 9 shows that the total consumer surplus increases with the deployment of femtocell service. This is mainly because users with low spectrum efficiency (small values of  $\theta$ ) will be

able to obtain high quality service by using femtocell and have better payoffs.

Nevertheless, some consumers could actually be worse off with dual services. Figure 10 shows users' surplus (payoff) with respect to their macrocell spectrum efficiency  $\theta$ . Here we let the total capacity  $B = 2.1$ . From Figure 9 we know that, in this case, femtocell deployment increases the total consumer surplus. However, Figure 10 shows that, for individual users, a user with small  $\theta$  obtains great payoff enhancement with femtocell deployment, whereas the payoff for a user with large  $\theta$  actually becomes worse off. The former is due to the benefit of service availability and quality improvement, while the latter is due to higher macrocell price.

Based on Figs. 8 to 10, we have the following result.

**Observation 2.** *After introducing femtocell service, the macrocell operator's profit increases since more users can be served now. Similarly, the total consumer surplus increases though some users' payoffs decrease. Overall, the total social welfare increases.*

## 5. Extension I: With Femtocell Operational Cost

In Section 4, we consider a model where femtocell service does not incur any additional cost comparing with the macrocell service. As shown in Figure 1, femtocell users' traffic will first go through users' home wireline broadband connections before reaching the control center of the cellular network. The broadband connection is owned by an Internet Service Provider (ISP). When the femtocell operator and the ISP belong to the same entity (*e.g.*, both belonging to AT&T) or the ISP is sharing-friendly (NetShare 2002), there is no additional cost for broadband access. Otherwise there is usually an access charge. In this section, we study the more general case where the ISP charges the femtocell operator fees for using the wireline Internet connection. We are interested in understanding how this operational cost affects the provision of femtocell service.

For simplicity, we assume that the total operational cost is linearly proportional to femtocell bandwidth with a coefficient  $C \in (0, 1)$ . This is motivated by the fact that femtocell traffics will use ISP's broadband resource, and many ISPs have adopted usage-based pricing (Deleon 2011). If  $C \geq 1$ , we can show that the femtocell operator will charge a femtocell price  $p_F > C \geq 1$ , and no user will choose the femtocell service (see (8)). Thus we will focus on the case where the linear coefficient  $C \in (0, 1)$ . The three-stage decision process is similar

to that shown in Figure 4. The analysis of Stage III is the same as in Section 4.3, and we hence focus on Stage II.

## 5.1 Stage II: Femtocell Operator's Spectrum Purchase and Pricing Decisions

In Stage II, the femtocell operator determines  $B_R$  and  $p_F$  to maximize its profit. We still use  $B_R^*(p_M, B_F)$  and  $p_F^*(p_M, B_F)$  to denote the equilibrium decisions of the femtocell operator.

By following a similar analysis as in Lemma 2, we obtain the following result.

**Lemma 5.** *At the equilibrium, the macrocell operator will satisfy all preferred demands from users with  $\theta \in [\theta_{th}^{pr}, 1]$ .*

The proof of Lemma 5 is very similar to Lemma 2 and is omitted. Based on Lemma 5, the femtocell operator will serve users with  $\theta \in [0, p_M/p_F)$ , and its profit is

$$\pi^{Femto}(p_F, B_R) = (p_F - C) \min \left( B_R, \int_0^{\frac{p_M}{p_F}} \left( \frac{1}{p_F} - 1 \right) d\theta \right) - p_M B_R. \quad (16)$$

we can explicitly write the femtocell operator's profit-maximization problem as

$$\begin{aligned} \max_{p_F \geq 0, B_R \geq 0} \quad & \pi^{Femto}(p_F, B_R) \\ \text{subject to} \quad & B_R \leq B_F, \\ & p_M + C \leq p_F \leq 1, \end{aligned} \quad (17)$$

where the second constraint shows that the femtocell price  $p_F$  should at least cover the total cost  $(p_M + C)$  for the femtocell operator. By solving Problem (17), we obtain the following result.

**Lemma 6.** *In Stage II, the femtocell operator's equilibrium femtocell price is*

$$p_F^*(p_M, B_F) = \max \left( \frac{2}{1 + \frac{1}{p_M + C}}, \frac{-p_M + \sqrt{(p_M)^2 + 4B_F p_M}}{2B_F} \right), \quad (18)$$

and its equilibrium femtocell bandwidth purchase is

$$B_R^*(p_M, B_F) = \min \left( p_M \frac{\frac{1}{(p_M + C)^2} - 1}{4}, B_F \right), \quad (19)$$

which equals users' total preferred demand in femtocell service. Then, users' preferred partition threshold equals users' finalized partition threshold (i.e.,  $\theta_{th} = \theta_{th}^{pr}$ ).

The proof of Lemma 6 is given in Appendix 2.

## 5.2 Stage I: Macrocell Operator's Spectrum Allocations and Pricing Decisions

Now let us study Stage I, where the macrocell operator determines  $p_M$ ,  $B_F$ , and  $B_M$  to maximize its profit. Let us denote its equilibrium decisions as  $p_M^*$ ,  $B_F^*$ , and  $B_M^*$ .

Notice that Lemma 5 shows that it is optimal for the macrocell operator to serve all users with  $\theta \in \left[\frac{p_M}{p_F^*(p_M, B_F)}, 1\right]$  by the macrocell service. By using the fact that  $B_M = B - B_F$ , we can eliminate variable  $B_M$ . The macrocell operator's profit is

$$\pi^{Macro}(p_M, B_F) = p_M B_R^*(B_F, p_M) + p_M \int_{\frac{p_M}{p_F^*(B_F, p_M)}}^1 \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta. \quad (20)$$

The macrocell operator's profit-maximization problem is

$$\begin{aligned} \max_{B_F, p_M} \quad & \pi^{Macro}(p_M, B_F), \\ \text{subject to} \quad & 0 \leq B_F + \int_{\frac{p_M}{p_F^*(B_F, p_M)}}^1 \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta \leq B, \\ & 0 < p_M \leq 1 - C, \end{aligned} \quad (21)$$

where  $B_R^*(B_F, p_M)$  and  $p_F^*(B_F, p_M)$  are given in (18) and (19), respectively. The second constraint shows that the total cost  $C + p_M$  to femtocell operator should be less than 1. Otherwise, the femtocell price  $p_F$  needs to be larger than  $C + p_M$  and thus larger than 1, and no user will subscribe to the femtocell service.

## 5.3 Numerical Results

Problem (21) is not convex and is difficult to solve in closed-form, but can be solved easily using numerical methods. Similar to Section 4, we can see that dual services degenerate to the macrocell service only benchmark when capacity is large. Here we will focus on how cost  $C$  will affect the division of two capacity regimes and the performance when capacity is small.

### 5.3.1 Impact of $C$ on Capacity Regime Boundary

Figure 11 illustrates how cost  $C$  affects the boundary between the low capacity and high capacity regimes. Recall that the boundary is 4.77 when  $C = 0$  (*i.e.*, Figures 5, 6, and 7). When  $C$  increases, the femtocell price  $p_F^*$  increases and demand for femtocell service decreases. This makes it less attractive to provide femtocell service. On the other hand,

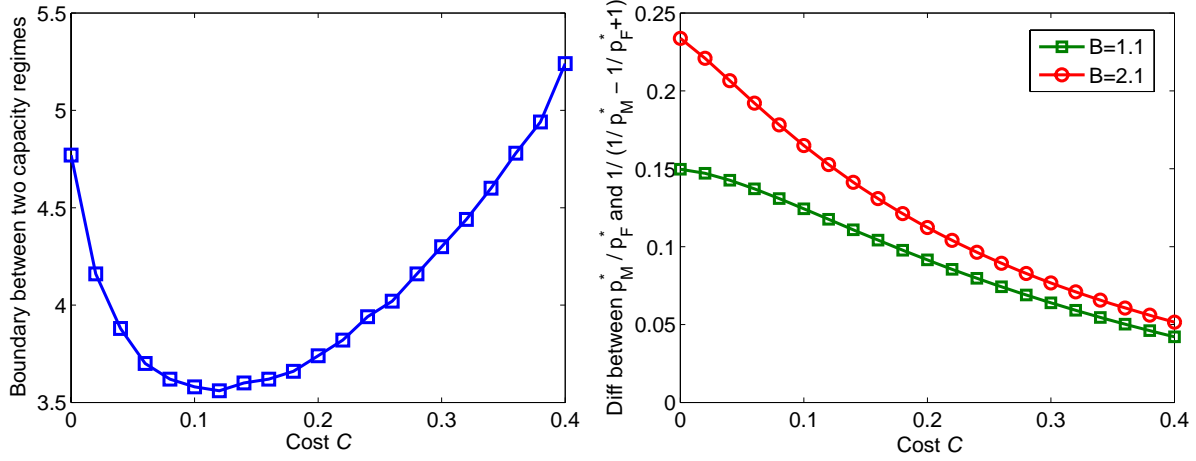


Figure 11: The boundary between low and high capacity regimes change with femtocell partition threshold  $\theta_{th} = \frac{p_M^*}{p_F^*}$  and macro-cell operational cost  $C$ .

Figure 12: The difference between users' preferred threshold  $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*} - \frac{1}{p_F^*} + 1}$  as a function of  $C$  and  $B$

the increase of price  $p_F^*$  also reduces the market competition, which makes the macrocell operator more willing to lease spectrum to the femtocell operator. The interactions of these two factors determine the boundary of the two capacity regimes. More specifically, with a small cost  $C \leq 0.12$ , the decrease of femtocell demands dominates and the boundary decreases. With a large cost  $C > 0.12$ , the decrease of competition dominates and the boundary increases. We will discuss these two factors in more details at a later point.

Figure 12 explicitly illustrates that a larger  $C$  decreases the gap between users' finalized partition threshold  $\theta_{th} = \frac{p_M^*}{p_F^*}$  and the threshold  $\tilde{\theta}_{th} = \frac{1}{\frac{1}{p_M^*} - \frac{1}{p_F^*} + 1}$  that the macrocell operator prefers, and thus makes the service competition less fierce. This gives more incentive to the macrocell operator to lease spectrum to the femtocell operator, which is the dominant factor that increases the boundary between two capacity regions when  $C$  increases (as shown in Figure 11).

**Observation 3.** *As cost  $C$  increases in femtocells, the femtocell operator has less incentive to provide femtocell service. However, the macrocell operator may benefit from the increase of  $C$  in terms of its profit since the service competition from femtocell operator become less intense.*

When cost  $C$  increases but is still small, we can show that femtocell price increases to compensate cost, and the macrocell operator will face the decrease of femtocell demands. In

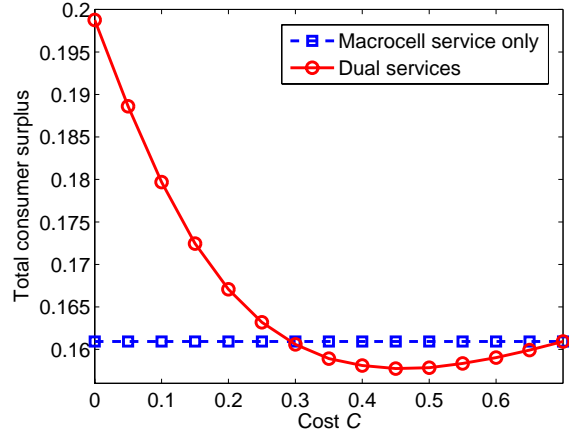
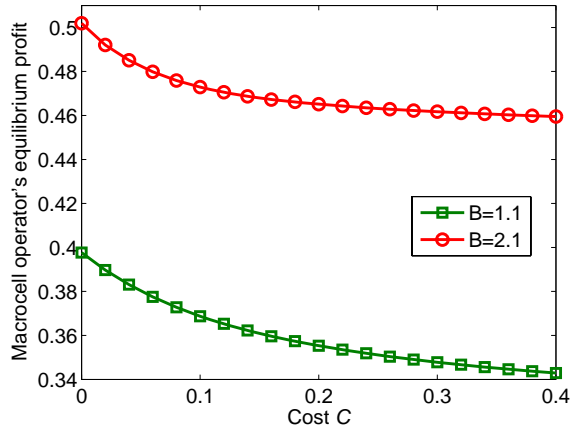


Figure 13: Macrocell operator's equilibrium profit as a function of cost  $C$  and capacity  $B$  plus between dual services and macrocell service only benchmark as functions of  $C$ . Here we fix  $B = 1.1$ .

this case, less femtocell band is needed and the macrocell operator's profit decreases in  $C$  (see Fig. 13). However, when cost  $C$  is large, competition between dual services mitigates and the macrocell operator allow the existence of femtocell service even for a large  $B$ . We can further observe from Fig. 11 that the macrocell operator still wants to lease bandwidth to femtocell operator even when  $B = 5.2$  under  $C = 0.4$ , while no femtocell service is provided under  $C = 0$  in this case. Thus the macrocell operator benefits from the high cost for large  $B$  in terms of its profit.

### 5.3.2 Impact of $C$ on Consumer Surplus and Social Welfare

In Figures 14 and 15, we investigate how the cost  $C$  affects total consumer surplus and social welfare. We focus on the low capacity regime only.

Figure 14 shows that the total consumer surplus is larger with dual services when  $C < 0.3$ , but is smaller with dual services when  $C > 0.3$ . In the latter case, femtocell users experience only small QoS improvements due to the high cost  $p_F^*$ , and macrocell users experience a  $p_M^*$  larger than  $p_M^{bench}$ . Note that macrocell price increases since all users can be served. As a result, the total consumer surplus decreases with dual services.

Figure 15 shows that social welfare is always larger with dual services for all possible values of  $C$ . Together with Figure 14, this shows that the macrocell operator obtains a larger profit by sacrificing the consumer surplus when  $C > 0.3$ .



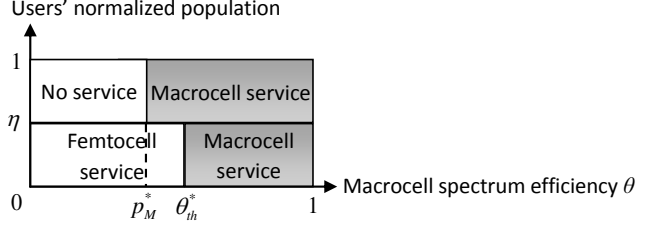
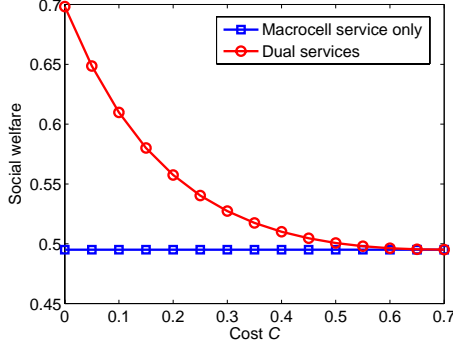


Figure 15: Comparison of social welfare between dual services and macrocell service only over space and macrocell spectrum efficiency as functions of cost  $C$ . Here the total capacity  $\theta$  is fixed at  $B = 1.1$

**Observation 4.** *After introducing femtocell service, total consumer surplus increases only when the cost  $C$  is small. The social surplus always increases.*

## 6. Extension II: With Limited Femtocell Coverage

In Section 4, we assume that femtocell service has the same ubiquitous coverage as the macrocell service. In this section, we look at the general case where the femtocell service only covers  $\eta \in (0, 1)$  portion of the user population, as illustrated in Figure 1. Then  $1 - \eta$  portion of users can only access the macrocell service. Figure 16 illustrates the users' possible service partitions over space and macrocell spectrum efficiency  $\theta$ . We call the  $\eta$  fraction users *overlapped users*, and the rest  $1 - \eta$  *non-overlapped users*. We are interested in understanding how the limited coverage affects the provision of femtocell service.

The three-stage decision process is similar to that depicted in Figure 4. The analysis of Stage III is the same as Section 4.3. Next we focus on Stage II.

Following a similar analysis as in Lemma 2, we can also conclude that overlapped users with  $\theta \in [\theta_{th}^{pr}, 1]$  will be served by macrocell service, and the other overlapped users will be served by femtocell service. That is,  $\theta_{th} = \theta_{th}^{pr}$ . Then we can similarly derive the following result.

**Lemma 7.** *In Stage II, the femtocell operator's equilibrium femtocell price is*

$$p_F^*(p_M, B_F) = \max \left( \frac{2}{\frac{1}{p_M} + 1}, \frac{-p_M\eta + \sqrt{(p_M\eta)^2 + 4p_MB_F\eta}}{2B_F} \right), \quad (22)$$

and its leased bandwidth from macrocell operator is

$$B_R^*(p_M, B_F) = \min \left( \eta p_M \left( \frac{\frac{1}{(p_M^M)^2} - 1}{4} \right), B_F \right), \quad (23)$$

which equals overlapped users' total preferred demand in femtocell service.

## 6.1 Macrocell Operator's Spectrum Allocations and Pricing in Stage I

Now we are ready to study Stage I, where the macrocell operator's profit-maximization problem is

$$\begin{aligned} \max_{p_M, B_F} \pi^{Macro}(p_M, B_F) &= p_M B_R^*(p_M, B_F) + p_M \int_{\frac{p_M}{p_F^*(p_M, B_F)}} \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta, \\ \text{subject to,} \quad 0 < B_F + \int_{\frac{p_M}{p_F^*(p_M, B_F)}} \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta &\leq B, \end{aligned} \quad (24)$$

where  $p_F^*(p_M, B_F)$  and  $B_R^*(p_M, B_F)$  are respectively given in (22) and (23).

## 6.2 Numerical Results

Problem (24) is not convex and is difficult to solve in closed-form, but can be solved easily using numerical methods. As in Sections 4.4 and 5.3, we can again clearly observe different behaviors in two capacity regimes: dual services degenerate to the macrocell service only benchmark in the high capacity regime. Unlike Section 5.3, the femtocell coverage  $\eta$  does not affect the boundary of the two capacity regions (*i.e.*, always at  $B = 4.77$ ). The two effects (QoS improvement and competition brought by femtocell service) coexist in  $\eta$  coverage.

We can show that as  $\eta$  increases, it is more attractive to provide femtocell service and the equilibrium femtocell (macrocell) band  $B_F^*$  ( $B_M^*$ ) increases (decreases). Yet both prices  $p_F^*$  and  $p_M^*$  increase in  $\eta$  (see Fig. 17). Intuitively, as  $\eta$  increases, more users are served with a larger total demand in femtocell service, which leads to a larger  $p_F^*$ . The overall wireless service (macrocell plus femtocell) become more efficient and the total user demand (of both services) will increase. Thus we can observe a larger  $p_M^*$ . Since the macrocell operator can sell total capacity in higher dual prices, its profit increases in  $\eta$ .

Figure 18 further shows that the total consumer surplus is larger with dual services than macrocell service only benchmark. This result is similar to Fig. 9 in Section 4.4.

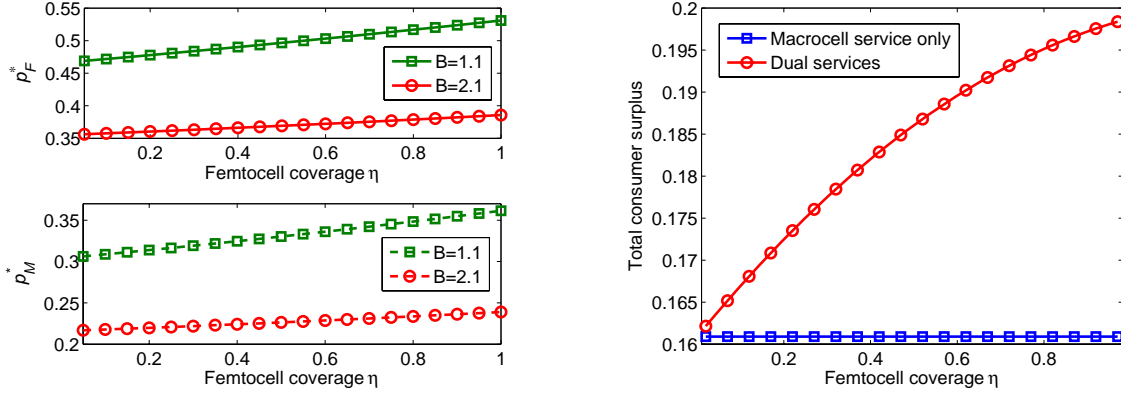


Figure 17: Equilibrium prices  $p_F^*$  and  $p_M^*$  as functions of  $\eta$  and  $B$

Figure 18: Comparison of total consumer surplus between dual services and macrocell benchmark under  $B = 1.1$ .

**Observation 5.** *As femtocell coverage expands, the overall wireless service (macrocell plus femtocell) becomes more efficient. Both macrocell operator's profit and total consumer surplus (as well as social welfare) increase in  $\eta$ .*

## 7. Conclusion

This paper studies the economic incentives for a macrocell operator to deploy new femtocell service in addition to its existing macrocell service. The femtocell service is provided by another party, the femtocell operator, who needs to lease the macrocell operator's capacity. We model the interactions among macrocell operator, femtocell operator, and users as a three-stage dynamic game, and derive the equilibrium capacity allocation and pricing decisions. Our analysis shows that the macrocell operator has an incentive to enable both macrocell and femtocell services when its total bandwidth is small, as femtocell service enhances user coverage and improves profits for both macrocell and femtocell operators. Notice that not all users will experience a payoff increase by the introduction of femtocell service in this case. However, when the total bandwidth is large, femtocell service becomes a severe competitor to macrocell service, and the macrocell operator thus has less incentive to lease its bandwidth to the femtocell operator. In this case, only macrocell service is provided to users.

Also, we further study the impact of operational cost of femtocell service. On one hand, we show that the increase of operational cost of femtocell service makes both operators' profits decrease. On the other hand, we show that the operational cost can mitigate femtocell operator's competition with macrocell operator. Finally, we investigate the impact of limited

femtocell coverage where only some users have access to femtocell service.

There are several directions to extend the results in this paper.

- We can further consider the “shared carriers” scheme besides “separate carriers”, where femtocell service and macrocell service share part of or the whole spectrum. We need to optimize the pricing and spectrum allocation decisions by trading off the increased spectrum efficiency and mutual interferences between macrocell and femtocell services.
- We can also consider the frequency spectrum reuse, where multiple femtocells can reuse the same spectrum if they do not overlap with each other in terms of coverage. In this case, femtocell service will become more attractive to the femtocell operator, as a single frequency band may support more users. However, frequency reuse might make the interference management complicated in areas where femtocells are densely deployed.
- We may also consider a more practical users’ models in dual services, by incorporating their heterogeneous levels of willingness to pay or sensitivity to achieved data rates.
- Moreover, we can extend our current monopoly case to oligopoly case, where multiple macrocell (femtocell) operators compete with each other. Intuitively, we can envision no provision of femtocell service only when all macrocell operators have adequate bandwidth. But the macrocell and femtocell prices will go down due to operators’ competitions.

## Appendix

### Appendix 1: Proof of Lemma 3

We first notice that the first term in the min operation of  $\pi^{Femto}(p_F, B_R)$  in (10) is increasing in both  $p_F$  and  $B_R$ , while the second term is decreasing in both  $p_F$  and  $B_R$ . Hence, the equilibrium  $p_F^*(p_M, B_F)$  and  $B_R^*(p_M, B_F)$  should make these two terms equal (*i.e.*, femtocell operator’s capacity equals users’ total preferred femtocell demand). Then we can write  $B_R^*(p_M, B_F)$  as a function of  $p_F$ , *i.e.*,

$$B_R^*(p_F) = \frac{(1 - p_F)p_M}{(p_F)^2}, \quad (25)$$

which should be no larger than  $B_F$  by a proper choice of  $p^F$ .

Then the femtocell operator's profit-maximization problem can be simplified from (11) to

$$\begin{aligned} \max_{p_F} \pi^{Femto}(p_F) &= \pi^{Femto}(p_F, B_R^*(p_F)) = \frac{p_M(1-p_F)(p_F-p_M)}{(p_F)^2} \\ \text{subject to} \quad &\frac{(1-p_F)p_M}{(p_F)^2} \leq B_F. \end{aligned} \quad (26)$$

Let us check the first derivative of  $\pi^{Femto}(p_F)$  over  $p_F$ , *i.e.*,

$$\frac{d\pi^{Femto}(p_F)}{dp_F} = p_M \frac{2p_M - (1+p_M)p_F}{(p_F)^3},$$

which is positive when  $p_F < 2p_M/(1+p_M)$  and is negative otherwise. Thus  $\pi^{Femto}(p_F)$  reaches its maximum when  $p_F = 2p_M/(1+p_M)$ . Also,  $p_F$  needs to satisfy the constraint in (26). Then the equilibrium femtocell price is

$$p_F^*(p_M, B_F) = \max \left( \frac{2p_M}{1+p_M}, \frac{-p_M + \sqrt{(p_M)^2 + 4B_F p_M}}{2B_F} \right).$$

By substituting this into (27), we obtain the equilibrium femtocell bandwidth purchase as

$$B_R^*(p_M, B_F) = \min \left( \frac{1 - (p_M)^2}{4p_M}, B_F \right).$$

## Appendix 2: Proof of Lemma 6

We can rewrite the femtocell operator's profit in (17) as

$$\pi^{Femto}(B_R, p_F) = \min(S_F(B_R, p_F), Q_F(B_R, p_F)),$$

where

$$S_F(B_R, p_F) = B_R(p_F - p_M - C)$$

and

$$Q_F(B_R, p_F) = \frac{p_M}{p_F} \left( \frac{1}{p_F} - 1 \right) (p_F - C) - p_M B_R.$$

It is clear that  $S_F(B_R, p_F)$  is increasing in  $B_R$  and  $p_F$ , and  $Q_F(B_R, p_F)$  is decreasing in  $B_R$ .

Let us consider the following two cases at the equilibrium:

- If  $S_F(B_R, p_F) > Q_F(B_R, p_F)$ , the femtocell operator's profit equals  $Q_F(B_R, p_F)$ . Then the femtocell operator will decrease  $B^R$ , which will decrease  $S_F(B_R, p_F)$  and increase  $S_F(B_R, p_F)$  until  $S_F(B_R, p_F) = Q_F(B_R, p_F)$ . The profit will be improved. Thus  $S_F(B_R, p_F) > Q_F(B_R, p_F)$  cannot be true at the equilibrium.

- Similarly, we can show that it is not possible to have  $S_F(B_R, p_F) < Q_F(B_R, p_F)$  at the equilibrium.

As a result,  $S_F(B_R, p_F) = Q_F(B_R, p_F)$  at the equilibrium, which leads to

$$B_R^*(p_F) = \frac{p_M}{p_F} \left( \frac{1}{p_F} - 1 \right). \quad (27)$$

The choice of  $p_F$  should satisfy that  $B_R^*(p_F) < B_F$ , *i.e.*,

$$p_F \geq \frac{-p_M + \sqrt{(p_M)^2 + 4p_M B_F}}{2B_F}. \quad (28)$$

Then the femtocell operator's profit-maximization problem in (17) can be simplified as

$$\begin{aligned} \max_{p_F} \quad & \pi^{Femto}(p_F) = \frac{p_M}{p_F} \left( \frac{1}{p_F} - 1 \right) (p_F - p_M - C), \\ \text{subject to} \quad & p_F \geq \frac{-p_M + \sqrt{(p_M)^2 + 4p_M B_F}}{2B_F}. \end{aligned} \quad (29)$$

The first order derivative of  $\pi^{Femto}(p_F)$  in (29) over  $p_F$  is

$$\frac{d\pi^{Femto}(p_F)}{dp_F} = \frac{p_M}{(p_F)^2} \left( (C + p_M) \left( \frac{2}{p_F} - 1 \right) - 1 \right),$$

which is positive when  $p_M < \frac{2}{1 + \frac{1}{p_M + C}}$  and negative otherwise. Thus  $\pi^{Femto}(p_F)$  achieves its maximum at  $p_F = \frac{2}{1 + \frac{1}{p_M + C}}$ . Also,  $p_F$  needs to satisfy the constraint in Problem (29). Hence, we obtain the equilibrium femtocell price  $p_M^*(p_M, B_F)$  in (18). By substituting (18) into (27), we can derive the equilibrium femtocell bandwidth request  $B_R^*(p_M, B_F)$  in (19).

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